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MILITARY GEODESY AND GEOSPACE SCIENCE
Faculty Material

Warren G. Heller
A. Richard LeSchack

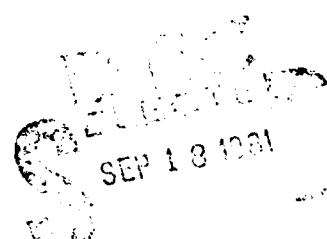
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AIR FORCE GEOPHYSICS LABORATORY
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This lecture course provides a full-year introduction to Military Geodesy and Geospace Science. Throughout the presentation a military perspective is maintained which links Mapping, Charting, and Geodesy (MC&G) issues with modern defense requirements. Elementary preparation is assumed in the subjects of general physics, mechanics, chemistry, aeronautics, and linear system theory. The student should also be familiar with differential equations, analytic geometry, and linear algebra. Some acquaintance with vector calculus is useful but not essential. | | |

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The topics covered herein are intended to provide conceptual rather than working knowledge. Ideally, the student completing this course will have attained a broad understanding of the MC&G field and will be able to develop specialized expertise quickly when required.

The notes are intended to be presented in chapter/section order within each of the four Units of Instruction. However, several of the subsections in these notes contain more advanced material which may be omitted without loss of continuity. These subsections are denoted with the symbol (†) after the title. A fifth volume contains faculty material.

The organizational flow of the lectures is from concepts in the initial sections, particularly in Unit One, to applications and specific systems later on. As a result the student is often referred ahead to provide motivation in regard to relevancy. In later chapters, however, the situation is reversed with the student referred back to review important conceptual material as necessary.

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MILITARY GEODESY AND GEOSPACE SCIENCE

FACULTY MATERIAL

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UNIT ONE

INTRODUCTION TO MAPPING, CHARTING, AND GEODESY (MC&G)

SUGGESTED LESSON PLANS*

| <u>Lecture</u> | <u>Material Covered</u> | <u>Textbook</u> |
|----------------|---|---------------------|
| 1 | Introduction and Orientation | 1.1 |
| 2 | Ellipsoid and Geoid | 1.2.1 |
| 3, 4, 5 | Surveying and Geodetic Positioning | 1.2.2 |
| 6, 7 | Photogrammetry | 1.2.3 |
| 8, 9 | Reference Systems (Earth-Fixed and Space-Fixed); Time | 1.2.4.1, 1.2.4.2 |
| 10 | Astronomic Positioning | 1.2.4.3 |
| 11, 12 | Geodetic Datums | 1.2.5 |
| 13, 14 | Map Projections | 1.2.6 |
| 15 | Inertial Survey Systems; Radio Interferometry | 1.2.7 |
| 16, 17 | Gravity and Geoid | 1.3.1, 1.3.2 |
| 18 | Gravimetry and Gradiometry | 1.3.3 |
| 19, 20 | Satellite Geodesy | 1.4 |

*Note: These lesson plans are based on 42-lectures/semester, with Units One and Two covered in the first semester; Units Three and Four, in the second.

UNIT TWO
WEAPON SYSTEMS AND MC&G

SUGGESTED LESSON PLANS

| <u>Lecture</u> | <u>Material Covered</u> | <u>Textbook</u> |
|----------------|---|------------------------------------|
| 21 | Introduction and Orientation | 2.1 |
| 22 ,23 | World Geodetic Systems | 2.2; also 1.2.5 (Unit One) |
| 24 ,25 | Introduction of Inertial Navigation, Mechanization of Inertial Navigation Systems | 2.3, 2.3.1 |
| 26, 27 | INS Errors | 2.3.2 |
| 28 | Effects of Gravity on INS | 2.3.3 |
| 29 | Gravity Models | 2.3.4; also 1.3.2 (Unit One) |
| 30-33 | Initialization of Inertial Systems | 2.3.5 |
| 34 | Launch Point Errors | 2.3.6 |
| 35-37 | Techniques for Aided Inertial Naviga- tion Examples | 2.3.7 |
| 38, 39 | Military Mapping and Charting | 2.4 |
| 40, 41 | Digital Data Bases | 2.5 |
| 42 | Mission Planning | 2.6 |

UNIT THREE
SOURCE DATA COLLECTION AND REMOTE SENSING

SUGGESTED LESSON PLANS

| <u>Lecture</u> | <u>Material Covered</u> | <u>Textbook</u> |
|----------------|---|-----------------|
| 43 | Introduction to Remote Sensing | 3.1 |
| 44, 45 | Photographic Systems | 3.2.1 |
| 46, 47 | Electro-optical Imaging Systems | 3.2.2 |
| 48 | Digital Image Characteristics; Non-Silver Materials | 3.2.3, 3.2.4 |
| 49 | Other Photographic Systems; Modula- tion Transfer Function | 3.2.5, 3.2.6 |
| 50 | Basic Imaging Operations | 3.3.1, 3.3.2 |
| 51 | Scanning | 3.3.3 |
| 52, 53 | Image Enhancement | 3.3.4 |
| 54, 55 | Atmospheric Effects | 3.4 |
| 56 | Properties of Infrared Radiation | 3.5.1 |
| 57 | Infrared Sensors | 3.5.2 |
| 58, 59 | Active and Passive IR Imagery | 3.5.3, 3.5.4 |
| 60 | IIR Scanning | 3.5.5 |
| 61 | Properties of MW and MMW Radiation | 3.6.1 |
| 62, 63 | Passive Sensors and Imagery | 3.6.2, 3.6.3 |
| 64 | Radar Sensors | 3.6.4 |
| 65 | Active Imagery; SAR | 3.6.5 |
| 66 | Target Positioning | 3.7 |

UNIT FOUR
SATELLITE SYSTEMS

SUGGESTED LESSON PLANS

| <u>Lecture</u> | <u>Material Covered</u> | <u>Textbook</u> |
|----------------|------------------------------------|------------------------|
| 67 | Introduction to Satellite Systems | 4.1 |
| 68, 69, 70 | Satellite Platform Considerations | 4.2 |
| 71 | Gravitational Perturbation Effects | 4.3 |
| 72, 73 | Attitude Reference and Control | 4.4.1, 4.4.2, 4.4.3 |
| 74 | Satellite Orbit Control | 4.4.4, 4.4.5 |
| 75 | Ground Tracking Systems | 4.5.1 |
| 76 | Satellite Altimetry | 4.5.2 |
| 77 | Satellite-to-Satellite Tracking | 4.5.3 |
| 78, 79, 80 | Navy Navigation Satellite System | 4.6 |
| 81, 82 | Global Positioning System | 4.7 |
| 83, 84 | Remote Sensing and LANDSAT | 4.8 |

UNIT ONE
REVIEW EXERCISES

Chapter Two
(Section 1.2.1)

1. At what latitude is the difference between geodetic and geocentric latitude a maximum?

A: $\Delta = (\phi - \phi') = (f + \frac{1}{2} f^2) \sin 2\phi - \frac{1}{2} f^2 \sin 4\phi \quad (1-1)$

$$\frac{d\Delta}{d\phi} = 2(f + \frac{1}{2} f^2) \cos 2\phi - 2f^2 \cos 4\phi \quad (1-2)$$

The extremum occurs when $\frac{d\Delta}{d\phi} = 0$.

$$2(f + \frac{1}{2} f^2) \cos 2\phi = 2f^2 \cos 4\phi \quad (1-3)$$

$$(1 + \frac{1}{2} f) \cos 2\phi = f \cos 4\phi \quad (1-4)$$

Let $x = \cos 2\phi$

$$\text{Then } \cos 4\phi = 2 \cos^2(2\phi) - 1 = 2x^2 - 1 \quad (1-5)$$

This leads to the quadratic equation:

$$(1 + \frac{1}{2} f)x = f[2x^2 - 1] \quad (1-6)$$

$$(1 + \frac{1}{2} f)x = 2fx^2 - f \quad (1-7)$$

$$(2f)x^2 - (1 + \frac{1}{2} f)x - f = 0 \quad (1-8)$$

$$x = \frac{(1 + \frac{1}{2} f) \pm [(1 + \frac{1}{2} f)^2 - 4(2f)(-f)]^{\frac{1}{2}}}{4f} \quad (1-9)$$

$$x = \frac{(1 + \frac{1}{2} f) \pm [1 + f + \frac{33}{4} f^2]^{\frac{1}{2}}}{4f} \quad (1-10)$$

For $f = 1/298.26$ (WGS 72)

$$x = 149.38 \text{ (extraneous root)} \text{ or } \underline{-0.013388}$$

If $x = \cos 2\phi = -0.013388$, then

$$2\phi = 90.767116 \quad (1-11)$$

$$\phi = 45.38 \quad (1-12)$$

At which value

$$\Delta = 0.19 \text{ deg} \quad (1-13)$$

Using the facts that $\Delta = 0$ at latitudes of 0 deg and 90 deg, that there is only one extremum, and that Δ is positive at the extremum, the student may easily demonstrate that this extremum is a maximum.

Of course, the student may prefer to solve Eq. (1-2) by an appropriate numerical method, in which case the extremum is verified to be a maximum by evaluation of Eq. (1-1) at points in the vicinity of the extremum.

(Section 1.2.2)

2. A triangulation survey starts with a baseline (running west to east) of 2000 m connecting points P_0 and P_1 . At point P_0 , the angle between the baseline and an unknown point, P_2 , is measured as 44 deg. At point P_1 , the angle between the baseline and point P_2 is measured to be 106 deg. Using P_0 as origin, what are the coordinates of point P_2 ?
- A: Using the formulas for plane triangles, it is found that P_2 lies 2671.00 m north of P_0 , 2765.90 m east of P_0 . The line P_0P_1 is 3845.05 m in length; P_1P_2 is 2778.63 m.
3. Continuing the triangulation survey of Exercise 2, the line P_1P_2 is used as a baseline to survey a new point, P_3 . At P_1 , the angle between the baseline (P_1P_2) and point P_3 is 52 deg. At P_2 , the angle between the baseline and P_3 is 88 deg. Find the coordinates of P_3 relative to the origin, P_0 .
- A: P_3 is 6005.57 m north and 1618.36 m east of P_0 . Line P_2P_3 is 3406.40 m; line P_1P_2 is 4320.15 m in length.
4. A trilateration survey starts with a baseline from the origin, P_0 , to an initial point, P_1 , with length 900 m and azimuth (measured positive from north toward east) of 30 deg. The following distances are measured:

| FROM | TO | MEASURED DISTANCE (m) |
|-------|-------|--------------------------|
| P_1 | P_2 | 1000 |
| P_0 | P_2 | 1100 |
| P_2 | P_3 | 1200 |
| P_1 | P_3 | 1800 |

Find the coordinates of P_1 , P_2 , and P_3 relative to the origin (P_0).

A: Using standard formulas of plane trigonometry:

| POINT | DISTANCE NORTH OF P_0 (m) | DISTANCE EAST OF P_0 (m) |
|-------|-----------------------------------|----------------------------------|
| P_1 | 779.42 | 450.00 |
| P_2 | 19.39 | 1099.83 |
| P_3 | 450.60 | 2219.68 |

5. An open traverse (refer to Fig. 1.2-9) begins at the origin, P_0 , along an initial baseline with azimuth 75 deg and length 1600 m, connecting P_0 with P_1 . The traverse continues as shown:

| FROM | TO | MEASURED DISTANCE (m) | MEASURED ANGLE (deg) |
|-------|-------|--------------------------|-------------------------|
| P_1 | P_2 | 2000 | 160 |
| P_2 | P_3 | 800 | 200 |
| P_3 | P_4 | 2500 | 250 |

Angles are measured in a clockwise sense from the previous traverse line. Find the coordinates of points P_1 , P_2 , P_3 , and P_4 relative to the origin (P_0).

A: From the formulas for a right triangle:

| POINT | DISTANCE NORTH OF P ₀ (m) | DISTANCE EAST OF P ₀ (m) |
|----------------|--|---|
| P ₁ | 414.11 | 1545.48 |
| P ₂ | 1561.26 | 3183.79 |
| P ₃ | 1768.32 | 3956.53 |
| P ₄ | 334.38 | 6004.41 |

6. Referring to Exercise 5, suppose that distance measurements are correct to within 0.1 percent, while angles are correct to within 1.0 min. Find upper bounds for the position error of point P₄.

A: The upper bound error analysis is based on the formulas for a right triangle:

$$x = s \cos \theta \quad (6-1)$$

and

$$y = s \sin \theta \quad (6-2)$$

To first order,

$$\delta x = (\cos \theta) \delta s - s \sin \theta \delta \theta \quad (6-3)$$

$$|\delta x| \leq |\cos \theta| |\delta s| + |s \sin \theta| |\delta \theta| \quad (6-4)$$

$$\delta y = (\sin \theta) \delta s + s \cos \theta \delta \theta \quad (6-5)$$

$$|\delta y| = |\sin \theta| |\delta s| + |s \cos \theta| |\delta \theta| \quad (6-6)$$

where:

s = measured distance

θ = measured angle

$\delta s, \delta \theta$ = errors in measured quantities

$\delta x, \delta y$ = errors in computed coordinates

Applying these formulas in succession to the four right triangles associated with the traverse gives these upper bounds:

| FROM | TO | NORTH ERROR (m) | EAST ERROR (m) |
|----------------|----------------|--------------------|-------------------|
| P ₀ | P ₁ | 0.86 | 1.67 |
| P ₁ | P ₂ | 1.62 | 1.97 |
| P ₂ | P ₃ | 0.43 | 0.83 |
| P ₃ | P ₄ | 2.03 | 2.47 |
| ----- | | | |
| P ₀ | P ₄ | 4.94 | 6.94 |

For a statistical approach to traverse errors, use the methods in Chapter Two of Introduction to Surveying by Mueller and Ramsayer.

(Section 1.2.3)

7. Describe the factors that limit the accuracy of maps prepared by photogrammetric techniques from overlapping aerial photographs.

A: To answer this question completely, the student should review all the steps in the process of map preparation (Section 1.2.3). Key sources of error that the student should identify are those associated with

- The camera (lens distortion, film plane flatness and stability, etc.)
- Aircraft position and attitude control
- Horizontal and vertical control survey errors
- Operator and equipment limitations in the stereocompilation process.

(Section 1.2.4)

8. It is stated in the text (Section 1.2.4.1) that "a perfect atomic clock would not, after a lapse of many years, correctly predict such phenomena as sunrise, sunset, star transits, eclipses, etc." Explain why this is the case.

A: Because these phenomena depend on earth rotation, which is nonuniform.

9. Using the Star Catalog section reproduced as Table 1.2-7 (Section 1.2.4.2), calculate the angular distance between the stars α CMa (Sirius) and γ Gem.

A: The spherical triangle to be solved has sides of

$$90^\circ - \delta_1 = 73.58^\circ$$

and

$$90^\circ - \delta_2 = 106.69^\circ$$

with an included angle of

$$\alpha_2 - \alpha_1 = 1.93^\circ$$

The third side is found most easily from the Law of Cosines, and is

$$d = 33.16$$

Chapter Three

10. If an individual weighs exactly 80 kg at sea level on the equator, how much would this person weight at the North Pole?

A: A correct answer requires an understanding of the distinction between weight and mass. The kilogram is a unit of mass, and an 80-kg person has the same mass anywhere. The term weight is a measure of the force exerted by gravity:

$$f = ma$$

where:

f = force in newtons (N)

m = mass in kg

a = acceleration in gravity in m/sec^2

Using 9.7803 m/sec^2 for the acceleration of gravity at the equator (Eq. 1.3-10, Chapter Three)

$$f = 782.42 \text{ N}$$

Since scales are calibrated in equivalent mass units, rather than force units, the force of 782.42 N is indicated as 80 kg. At the pole (Eq. 1.3-10), the acceleration of gravity is 9.8321 m/sec^2 ; hence

$$f = 786.57 \text{ N}$$

and the scale would read

$$m = 80.42 \text{ kg}$$

The apparent excess weight (in mass units) is 420 g (nearly 1 lb).

11. If the earth were to stop rotating, by how much would gravity at the equator change?

A: The centrifugal acceleration is

$$a = \omega^2 r \cos^2 \phi'$$

with

$$\omega = 7.3 \times 10^{-5} \text{ rad/sec (earth's rotation rate)}$$

$$r = 6.378 \times 10^5 \text{ m (earth's equatorial radius)}$$

$$\phi' = 0 \text{ (latitude)}$$

Hence

$$a = 0.034 \text{ m/sec}^2$$

This is about 0.35 percent of the nominal sea level value.

12. Compute the excess gravity acceleration (gravity anomaly) at the surface of the earth, caused by a sphere of depleted uranium, with a mass of 100 kg, buried 10 m below the surface. Would the presence of this object be detectable by the use of ordinary gravimeters?

A: Because gravity accelerations are additive, the excess acceleration (anomaly) directly above the object can be computed as the acceleration due to the buried sphere alone, ignoring the attraction of the earth. From Appendix A (Eq. A.1-6), the acceleration is

$$a = -\frac{GM}{h^2}$$

where:

$$G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{sec}^2$$

$$M = 100 \text{ kg}$$

$$h = 10 \text{ m}$$

since the acceleration due to a uniform sphere is the same as that due to a point mass located at the center of the sphere. (The fact that the sphere is uranium is, of course, irrelevant.) Therefore,

$$a = - \frac{(6.67 \times 10^{-11})(100)}{(10)^2} = -6.67 \times 10^{-11} \text{ m/sec}^2$$

Since $10^{-5} \text{ m/sec}^2 = 1 \text{ mgal}$,

$$a = -6.67 \times 10^{-6} \text{ mgal}$$

The excess acceleration would not be measurable.

13. Referring to Appendix B as required, verify Fig. 1.3-4b.

A: Using Eq. 1.2-16 with $n=2$; $m=2$ gives

$$\begin{aligned} Y_{221} &= P_{22}(\sin \phi') \sin 2\lambda \\ &= 3 \cos^2 \phi' \sin 2\lambda \end{aligned}$$

The variation along a meridian (for example, $\lambda = 45$ deg) is then as shown in Fig. 1.3-4b.

14. Referring to Appendix B as required, plot the Legendre polynomial $P_3(\sin \phi')$ and verify Fig. 1.3-4a.

$$\begin{aligned}
 A: \quad P_3(\sin \phi') &= \frac{5}{2} \sin^3 \phi' - \frac{3}{2} \sin \phi' \\
 &= \frac{1}{2} \sin \phi' (5 \sin^2 \phi' - 3)
 \end{aligned}$$

The student should sketch a plot of this function between $\phi' = -90$ deg and $\phi' = 90$ deg, showing three zero-crossings and four extrema. This plot is then associated with the variation of the spherical harmonic ($n=3$; $m=0$) along a meridian.

(Section 1.3.2)

15. What is the normal gravity at latitude 48 deg N using the International Gravity Formula [Eq. (1.3-5)]?

A. 980899.78 mgal

16. What is the normal gravity at latitude 48 deg N using the Geodetic Reference System 1967 gravity formula?

A: 980890.15 mgal

17. At a point at sea level at latitude 48 deg N, there is a measured gravity of 980933.8 mgal. What is the gravity anomaly at that point?

A: $\Delta g = g - \gamma$

34.0 mgal International Gravity Formula

43.6 mgal GRS 67

(Section 1.3.3)

18. Suppose that a gravity measurement is to be made using a pendulum. If the pendulum's period is to be measured very precisely, how accurately must the dimensional stability of the pendulum's length be maintained for measurement accuracy of one mgal? one μ gal?

A: Pendulum's period $T = 2\pi\sqrt{l/g}$. Hence $g(\text{measured}) = \frac{4\pi^2 l}{T^2}$. The nominal value of g is 980 gal. Thus one mgal accuracy requires $10^{-3}/980 \approx$ one part in one million dimensional stability. One microgal accuracy requires one part in a billion.

19. In Exercise 18, what is the timing accuracy required to be commensurate with a one mgal level of dimensional stability.

A: Perturbing the equation

$$g(\text{measured}) = \frac{4\pi^2 l}{T^2} \text{ gives}$$

$$\Delta g(\text{measured}) = \frac{4\pi^2 \Delta l}{T^2} - \frac{8\pi^2 l}{T^3} \Delta T$$

For the two error terms to be of the same magnitude, it is necessary that

$$\frac{4\pi^2 \delta l}{T^2} = \frac{8\pi^2 l \Delta T}{T^3}$$

Simplifying gives

$$\frac{\Delta l}{l} = 2 \frac{\Delta T}{T}$$

One mgal accuracy requires $\Delta l/l$ to be one part per million. The timing uncertainty for one mgal of timing-induced error is therefore one-half part per million.

20. Suppose that a pendulum gravimeter has both a one part per million (RMS) dimensional error and an RMS timing error as computed in the previous exercise. If both of these errors are random and are unrelated, what is the RMS error in measured gravity?

- A: Independent error sources add by summing the squares of the RMS values of each error source. The square root of the sum is the RMS output error. Both length and timing errors contribute one mgal. Thus

$$(1.0^2 + 1.0^2)^{\frac{1}{2}} = 1.4 \text{ mgal RMS}$$

UNIT TWO
REVIEW EXERCISES

Chapter Three

1. Must the integrations illustrated in Fig. 2.3-1 be mechanized in an inertial reference frame? Discuss.

A: No, any convenient reference frame is suitable, but if it is non-inertial, the body forces acting on the vehicle must be accounted for in the course of the mechanization.

2. Why is it not possible to use an inertial system's accelerometers to measure and subtract away the effect of gravity?

A: The accelerometer measurement cannot distinguish between gravity and vehicle acceleration. This is a fundamental physical law and cannot be altered by improved accelerometers.

3. A locally-level mechanized inertial system provides navigation information for an aircraft flying at constant altitude along a meridian at 800 km/hr. Neglecting earth rotation, what is the angular slew rate of the inertial platform with respect to inertial space? (Hint: Consider the angle between the platform-indicated vertical and the earth's spin axis.)

A: The platform maintains itself with respect to the local vertical as the aircraft moves. Thus the platform's angular rate with respect to the earth's center is the same as the aircraft's rate. Since the motion is along a meridian, the angle between the earth's spin axis and the platform-indicated vertical changes as

$$\dot{\phi} = \frac{v_{aircraft}}{R_{earth}} \text{ rad/hr}$$

Using 6.4×10^3 km as an approximate value for the equatorial radius of the earth, the slew rate is

$$\dot{\phi} = \frac{800 \text{ km/hr}}{6.4 \times 10^3 \text{ km}} = 0.13 \text{ rad/hr}$$

4. Consider a vehicle with a local-level-mechanized inertial system which is stationary at the equator, at a place where each component of the deflection of the vertical is 10 sec and the vertical gravity disturbance is 50 mgal . Suppose that the position error is one km in the two level channels and 3.05 m in the vertical. Also suppose that the gyro errors have results in ψ angle errors of 5 sec in all three channels. The accelerometer errors are $5 \times 10^{-6} \text{ g}$ for each accelerometer (where the g is a unit of acceleration equal to nominal sea-level gravity -- 9.8 m/sec^2). Tabulate the value of each term in Eqs. 2.3-11 through 2.3-12. Note that these terms correspond to the signals entering the acceleration summing nodes in Fig. 2.3-9. Use units of 10^{-6} g . What does this example illustrate about the cross-coupling terms of Fig. 2.3-9?

| A: | ACCELERATION MODEL ERROR (μg) | $\frac{d}{dt} (\delta v_N)$ | $\frac{d}{dt} (\delta v_N)$ | $\frac{d}{dt} (\delta v_2)$ |
|---|--|---|--|-----------------------------|
| <u>Individual Terms</u> | | | | |
| Accelerometer Errors (μ_N , μ_E , μ_Z) | 5 | 5 | 5 | |
| Gyro Errors* ($-\dot{\theta}\psi_E$, $-\dot{\phi}\psi_N$) | -95 (4.9 $\mu\text{sec/sec}$) | -25 | 0 | |
| Position Errors ($-\frac{g}{R} \delta p_N$, $-\frac{g}{R} \delta p_E$, $-\frac{2g}{R} \delta h$) | 153 | 153 | 306 | |
| Gravity Field Errors (σg_N , δg_E , δg_Z) | 47 (4.9 $\mu\text{sec/sec}$) | 48 | 51 ($0.98 \text{ mgal}/10^{-6} \text{ g}$) | |
| Velocity Error** ($-2\Omega \sin L \delta v_E$, $2\Omega \sin L \delta v_N + 2\Omega \cos L \delta v_Z$, $-2\Omega \cos L \delta v_Z$) | 5 | 0.4 ($\Omega \approx 15 \text{ deg/hr}$) | 0.4 | |

$$*A_N = A_E = A_Z = 0$$

$$**v_N = v_E = v_Z = 0$$

5. Suppose an inertial system uses a GM/r^2 gravity model instead of a reference ellipsoid gravity model. If the nominal surface value of gravity is 978.049 gal, use the approximate ellipsoidal gravity formula.

$$g = 978.049(1 + 0.0053 \sin^2 \phi) \text{ gal}$$

to compute the vertical acceleration error sensed by the system at a latitude of 45 deg. At this rate how long does it take for 100 m of altitude error to accumulate? Assume that the vertical gravity disturbance is zero at the location of the system.

A: The mechanized value of gravity is 978.049 cm/sec^2 . The ellipsoidal value of g at 45 deg is 980.641 cm/sec^2 . The difference between the two is 2.6 cm/sec^2 . Height error grows as $\frac{1}{2} (2.6)t^2$. One hundred meters of error is reached after 88 seconds.

6. Suppose a guidance system is to be compensated for gravity by using a model of the form of Eq. 2.3-20. What error sources might be associated with such a model if the maximum value of n is 20.

A: Accuracy of the value of GM, accuracy of C_{nm} and S_{nm} coefficients (errors of commission), and the contributions of the neglected terms beyond degree 20 (errors of omission).

(Section 2.3.5)

7. In a ballistic missile application it is desired to erect the inertial guidance platform with the accelerometer sensing axes symmetrically distributed about the local vertical (gravity vector) and with one accelerometer sensing axis pointing downrange. Assuming the use of a mutually orthogonal set of accelerometers, what are the nominal (error-free) accelerometer outputs (f_x , f_y , and f_z) when alignment is complete? (Hint: Use Eq. 2.3-23 and symmetry considerations.)

A: $f_x = f_y = f_z = 0.58 \text{ g.}$

8. A local level inertial platform is aligned in a stationary vehicle with the (orthogonal) x and y sensor axes in the level plane at a point on the earth's surface at which the geodetic and astronomic verticals coincide (zero deflections of the vertical) and the common latitude is ϕ deg. The x gyro axis points 5 deg north of east. The gyro torquing rate commands from the navigation computer are given by:

$$\omega_x = 0$$

$$\omega_y = \Omega \cos \phi$$

$$\omega_z = \Omega \sin \phi$$

where Ω = Earth Rotation Rate relative to Inertial Space.

What are the observed platform tilt rates ($\dot{\phi}_x$ and $\dot{\phi}_y$) about the platform x and y axes respectively? (Hint: Compute the components of earth rate appearing about the actual (misaligned) sensor axes (Ω_{xH} and Ω_{yM}) and use Eq. 2.3-27 to get tilt rates.)

A: $\dot{\phi}_x = \Omega \cos \phi \sin 5^\circ$

$$\dot{\phi}_y = \Omega \cos \phi (1 - \cos 5^\circ)$$

9. Using the small-angle approximations, $\sin 5^\circ \approx 0.1$ and $\cos 5^\circ \approx 1.0$, evaluate the tilt rates $\dot{\phi}_x$ and $\dot{\phi}_y$ derived in Problem 8 for vehicle latitudes $\phi_1 = 30$ deg N and $\phi_2 = 70$ deg N. Answer the following questions:

- (1) Which accelerometer senses a tilt rate that can be used to estimate and/or control the platform azimuth misalignment?
- (2) What is the change in the magnitude of the gyrocompassing control signal as the alignment latitude increases?
- (3) What does this difference tell you about inertial platform self-alignment capabilities at high latitudes in the presence of accelerometer time-varying errors or gyro bias drift errors?

A: At latitude ϕ_1 : $\dot{\phi}_x = 0.087 \Omega$ and $\dot{\phi}_y = 0$

At latitude ϕ_2 : $\dot{\phi}_x = 0.034 \Omega$ and $\dot{\phi}_y = 0$

- (1) The y-Accelerometer. Tilt rates appear about the x-axis only.
 - (2) The control signal ($\dot{\phi}_x$) decreases with increasing latitude.
 - (3) The decrease in the control signal with increasing latitude implies increasing difficulty in reaching a satisfactory azimuth alignment solution in the face of accelerometer and gyro errors. The "signal-to-noise" ratio decreases at higher latitudes. This is a manifestation of approaching collinearity between earth spin rate and local gravity vectors.
10. An error-free, north-slaved, local level inertial platform is self-aligned in a stationary vehicle at a point on the earth's surface at latitude 30 deg N. An easterly deflection of the vertical, $\eta = 10 \text{ sec}$, exists at the alignment location. The existence of this deflection of the vertical is not accounted for in the navigation computer. Evaluate the effect of this omission on:
- (1) Platform tilt errors
 - (2) Platform azimuth error

relative to the (geodetic) navigation coordinate frame when alignment is complete. Repeat the evaluation when the vehicle latitude is 70 deg N and the same conditions exist.

- A: (a) $\phi_{y0} = 10$ sec; $\phi_{x0} = 0$; Azimuth error, $\psi_{z0} = 5.8 \text{ sec}$
(b) $\phi_{y0} = 10$ sec; $\phi_{x0} = 0$; Azimuth error, $\psi_{z0} = 27.5 \text{ sec}$

Equations 2.3-28 gives azimuth errors.

11. An aircraft is equipped with an avionics suite that includes a gimballed inertial platform, a Doppler radar set and a LORAN receiver. Two distinct and separate modes of in-flight initialization are mechanized in the system:

- (1) Doppler (Velocity) - Inertial
- (2) LORAN (Position) - Inertial.

In the former mechanization the Doppler velocity inputs are transformed directly into inertial sensor coordinates, using resolvers mounted between the gimbals of the inertial platform, prior to comparison with the inertial system velocity outputs. In the latter mechanization the LORAN receiver outputs are converted to geodetic latitude and longitude in the navigation computer and compared with the latitude and longitude outputs of the inertial system to form alignment control signals.

- (a) What is the basic difference in the dynamics of the inertial platform alignment process between the two mechanizations?
- (b) Can a complete inertial system initialization be achieved if the LORAN receiver is inoperative?

- A: (a) In the second mechanization, azimuth alignment varies with aircraft flight path (spatial rate gyrocompassing).

(b) Not unless an additional method for discrete or continuous position updating is available.

(Section 2.3.6)

12. It is 1985! USAF is considering the deployment of a new ballistic missile system as an augmentation to the aging Minuteman force. In order to obstruct a surprise attack by opposing forces, this new missile will be deployed in truck-like transports which will roam at random through the deserts of southwestern CONUS. The missiles will be erected and fired from the moving transporter when the order to launch is received.

A weapon system CEP of 75 m is desired, and an allowance for the guidance system contribution to this total has been set at 30 m, including the effects of launch site uncertainties. The latest generation of inertial guidance hardware indicates that there will be no problem in meeting this accuracy goal if the initial condition errors can be controlled. The transporter has accordingly been equipped with special radio devices that allow it to determine its position continuously to an accuracy of about 3 m. Has the initial condition problem been fully addressed? Discuss.

A: No. The miss sensitivities listed in Table 2.3-1 show that suppressing the launch point position error to a level of 3 m is not sufficient. The velocity error also will have to be constrained to the order of 10^{-2} m/sec if the accuracy goal is to be achieved!

(Section 2.3.7)

13. Why are aided inertial navigation techniques preferred over unaided inertial navigation techniques? Give examples of the two types of Aided INS errors, and how they are counteracted.

A: Errors in an unaided INS typically grow with time in an unbounded manner. To maintain accuracy, the INS used in modern aircraft, missiles, and submarines are normally aided or "damped" with data from external (i.e., non-inertial) sources.

The two types of Aided INS errors are those 1) associated with the measurement device itself and referred to as instrument errors and those 2) induced by external factors which operate independently of instrument quality and referred to as environmental errors.

Examples of 1) are gyro drift, accelerometer bias, or radio receiver noise.

Example of 2) are radio-wave propagation anomalies and barometric variations due to weather patterns.

14. What is the fundamental technique behind terrain-matching navigational methods? Give examples.

A: Terrain matching is a navigational process based on recognition of terrain features. This is fundamentally a differencing technique, where position information is inferred by comparing certain measured terrain features (or possibly man-made objects) with reference features stored in some type of map.

One example is a pilot flying under VFR who establishes his position by comparing observed ground features with those indicated on his navigator's map.

Another example is photographic correlation schemes of map matching. With these schemes, the terrain is observed photographically and compared with photographically-based prestored reference maps until some best correlation of images is achieved between the observation and the reference.

15. Autonomous terminal-homing guidance systems generally use scene-matching concepts to determine the location of the vehicle or its position relative to the target. Give three specific, different difficulties that can be encountered in preparing and using the reference imagery model by the vehicle's onboard navigation system for correlation guidance.

A: The most common form of course data, from which each mission-dependent reference map is prepared, is stereoscopic optical photography from high-altitude aircraft or satellites.

One problem is that the original scene viewed in the initial photography may have to be regenerated as if viewed by a different sensor operating at a different wavelength, in order to match the image expected from the sensor aboard the autonomously-guided vehicle.

Another problem is the difficulty in predicting imagery at the same wavelength. For example, imagery of a building complex, obtained with a high-resolution, thermal IR scanner, is dependent in part on such unknown factors

as internal building structure, heating/air conditioning of the buildings during the period prior to imaging, recent solar illumination history, cloud cover, surface conditions, fog/haze/smoke densities, etc.

A third problem is that the scene correlation algorithms must be insensitive to image distortion arising from variations in perspective.

16. Suppose that a local-level mechanized inertial platform points toward a star located at the zenith. Discuss the improvement in system azimuth error that can be gained from this measurement.

A: Refer to Fig. 2.3-24. The star tracker's barrel axis is vertical, thus the measurement provides no ψ angle azimuth information.

17. Consider an inertial system which has no platform misalignment (i.e., $\theta = 0$ in Fig. 2.3-24). Can any error-free star fix correct all other system errors? Discuss.

A: The star fix provides ψ angle information in only two coordinates. The component of ψ along the line of sight to the star is not improved.

Chapter Four

18. How can Point Positioning Data Bases (PPDBs) be used to support tactical bombing operations?

A: Targets identified in reconnaissance photographs can often be located approximately with respect to nearby landmarks. The positions of the landmarks in an appropriate coordinate system can be determined using the PPDB

photochips and the APPS. The target location can then be determined in the chosen coordinate system.

Chapter 5

19. What are the key considerations for selection of a navigation/guidance concept based on an on-board reference map?
 - A: (a) Available on-board computing power and memory volume
 - (b) Sensor type(s) and resolution
 - (c) Reference data representation selection.
20. What are the principal sources of navigation errors which arise from the use of an on-board reference map?
 - A: (a) Map source material resolution, collection geometry, and ground control
 - (b) Translation from source to reference map data, especially if source is based on a different sensor
 - (c) Temporal variations in sensed data for the given terrain.
21. (a) What are the principal tasks to be accomplished in preparing reference maps for specific missions?
 - A: (a) Scene content identification
 - (b) Reference effectiveness prediction
 - (c) Scene selection.

(b) What implications do these have for the tactical or strategic mission planner?

- A: (a) Process is labor-intensive/special skills required
(b) Extensive computer support required
(c) Over-specification may result in slow response, overload of on-board system
(d) Error budget must be kept in mind.

22. What are the elements of an observable feature in a terrain data base?

- A: (a) Lack of ambiguity relative to neighboring features
(b) Minimal measurement uncertainty with respect to on-board sensor.

Chapter Six

23. Give the three important interdependent performance criteria, as related to route analysis for low-altitude, high-speed weapon delivery vehicles such as cruise missiles penetrating enemy-controlled areas.

A: The important performance criteria are 1) probability of detection (being neutralized by the enemy), 2) range loss, and 3) probability of crashing (clobber).

24. Refer to Eqs. 2.6-5 and 2.6-6. Assume that a particular cruise missile flies with a standard deviation of error in achieved clearance altitude (σ_{ACL}) of 25 ft, and a

standard deviation of the clearance rate (σ_{ACL}) of 5 ft/sec. What is the lowest mean flight altitude that can be achieved, for a 2-hour flight, that results in at most a 1% probability of clobber?

A:

$$P_{clobber(1)} = 0.01 = 1 - e^{-\lambda T} = 1 - e^{-7200\lambda}$$
$$1.39588 \times 10^{-6}/\text{sec} = \lambda$$

$$1.39588 \times 10^{-6} = \lambda = \frac{1}{2\pi} \frac{\sigma_{ACL}}{\sigma_{ACL}} \exp - \frac{A_M^2}{2\sigma_{ACL}^2}$$
$$10.035 = \frac{A_M^2}{2\sigma_{ACL}^2}$$

$$\text{Mean Flight Altitude } A_M = 112 \text{ Feet.}$$

UNIT THREE
REVIEW EXERCISES

Chapter One

1. Define the term remote sensing and give several examples.

A: As defined in Section 3.1.1, remote sensing refers to techniques for obtaining information about objects, areas, or phenomena by analyzing data collected by devices located at a distance from the object of investigation. In the examples discussed in the text, remote sensing is based on electromagnetic energy that is either emitted or reflected by the objects under study. This energy may be visible light, ultraviolet, infrared, or microwave.

Chapter Two

2. Briefly summarize the principal characteristics, advantages, and disadvantages of the two major imaging technologies -- photochemical and photo-electronic.

A: The student's answer should include the information contained in Table 3.2-1, as well as in the introduction to Chapter Two.

3. It is frequently stated that fluorescent light sources are more efficient than incandescent devices. Explain the meaning of the statement.

- A: Efficiency, in this context, is measured by the lumen-to-watt ratio of the device. For fluorescent lights the ratio can be as high as 80 lumens/watt, while a typical figure for incandescent light is 10 lumens/watt. The incandescent lamp dissipates about 20 percent of the energy supplied to it in the form of heat conducted through the socket or convected into the surrounding air. The remaining 80 percent is radiated in the form of electromagnetic energy, but most of this energy (about 85 percent) is in the infrared. The fluorescent bulb and its associated circuitry produce considerably less heat; also, the spectral distribution of the light emitted by the bulb is better matched to the relative spectral efficiency of the human eye (Fig. 3.2-2). Nonetheless, compared to the theoretical maximum of 686 lumens/watt, neither type of device would be considered efficient.
4. What would be the color of the light from a hypothetical device that produces light from electricity at close to the theoretical optimum efficiency of 686 lumens/watt?
- A: Such a hypothetical device would emit monochromatic light at the wavelength of maximum spectral efficiency for the human eye, about 550 to 560 nm (green). Refer to Fig. 3.2-2.
5. On a very bright day when the sky is clear, the illuminance due to sunlight may be as high as 10^4 footcandle. Express this value in metric units.
- A: The correspondence is given in Table 3.2-3. The illuminance due to sunlight would be about 10^5 lux (lumen/m^2).

6. Although silver fluoride belongs to the class of compounds called silver halides, it is not mentioned in most discussions of silver-based photographic technology. Explain.

A: Because silver fluoride is highly soluble in water, it is not usable in most photographic applications.

7. What is a latent image in photography?

A: The student's reply should summarize the relevant discussion in Section 3.2.1. The key sentence (page 3-19): "The distribution, within the emulsion, of altered and unaltered halide microcrystals, in accordance with the light intensities of the image, is called the latent image."

8. Assume that it is necessary to examine an exposed, but undeveloped, strip of aerial film to obtain information about the latent images contained thereon. What are some ways in which this can be done?

A: It is expected that the student will state unequivocally that a latent image can be detected only by developing it.

9. For the H and D curve shown in Fig. 3.2-5, estimate the value of gamma.

A: As the curve is drawn, gamma is about 1.33, somewhat higher than the usual range of values for pictorial or aerial photography.

10. Explain the reciprocity law and define the term reciprocity failure.

A: The reciprocity law is an approximation which states that the photographic effect (as measured by the density resulting from standard development) is a function of the product of intensity and exposure time (lux-seconds), and does not depend on the individual values of these two quantities, so long as the product is the same. For most pictorial and aerial films, there is a fairly wide range of illuminance levels for which the approximation gives satisfactory accuracy. At very low or very high illuminance levels, the film characteristics are not described well by the reciprocity law. This phenomenon is called reciprocity failure.

11. Why is reciprocity failure a more serious problem with color film than with black and white materials?

A: The three emulsion layers in color film typically have different sensitivity and reciprocity characteristics. Consequently, reciprocity failure affects color balance. The resulting color shifts can sometimes be compensated for by the use of filters (if the illuminance deviates only slightly from the design range), but severe reciprocity failure generally leads to unusable results.

12. Distinguish between photoconductive and photoemissive devices.

A: Both types of devices convert light signals to electrical signals, but they exploit different physical principles. In a photoconductive device, the light energy decreases the electrical resistance of a suitable semiconductor material. Thus the current in a circuit including such a device is modulated in accordance with changes in light intensity. Photoemissive devices, on the other hand,

depend on the photoelectric effect, in which incident light causes the ejection of electrons from the surface of the photosensitive material. These photoelectrons are collected to produce an electrical signal.

13. As a general rule, there is one characteristic that distinguishes all of the non-silver photographic processes from conventional silver-based technology. Explain.

A: The distinguishing characteristic is photographic speed, or sensitivity to light. Conventional materials can be made far more sensitive to light than any non-silver material.

14. Explain what is meant by the modulation transfer function (MTF).

A: The student may answer in informal terms (pages 3-73 to 3-75) involving the degradation of the image of a bar pattern test object; or in formal terms (pages 3-76 to 3-82), as the absolute value of the normalized Fourier transform of the point spread function (PSF).

Chapter Three

15. Distinguish between the concepts of sampling and digitizing.

A: The process of sampling breaks up a continuous picture into a finite number of discrete picture elements (pixels). Each pixel is characterized by an average value of a physical variable like light intensity or optical density. The process of digitizing (or quantizing) divides the range of values of the physical variable into a discrete number of steps (the number of steps is usually chosen to be a power of two).

16. The image enhancement techniques of Chapter Three are described in terms of sampled and digitized images available in digital form (for computer storage and manipulation). These techniques are, therefore, directly applicable to imagery produced by electro-optical devices. How can they be applied to images in continuous form on photographic film?

A: By sampling, digitizing, and storing the film image, using the methods of Sections 3.3.2 and 3.3.3. It may be of interest that there are certain techniques that can be applied directly to film images without the use of a computer. These methods are purely photochemical in nature, and are most effective for gray scale manipulation, contrast enhancement, and edge sharpening. While they do not require the use of scanning, digitizing, and computing equipment, they do require highly trained personnel and darkroom facilities.

Chapter Four

17. This exercise refers to Figs. 3.4-1a, b and 3.4-2. For each of the environmental/weather conditions listed below, and for a fixed aperture size, specify which of the atmospheric windows allows ≤ 2 dB/km total one-way attenuation while maximizing the angular resolution obtainable.

- Atmospheric Windows:
- a. Visible light
 - b. Mid-IR ($3\text{-}5 \mu\text{m}$)
 - c. Far-IR ($8\text{-}14 \mu\text{m}$)
 - d. 220 GHz
 - e. 140 GHz
 - f. 94 GHz
 - g. 35 GHz
 - h. < 20 GHz

Weather Conditions:

1. Fog, 400-m visibility
2. Haze, 1.2-km visibility
3. Fog, 100-m visibility
4. Moderate rain (0.4 cm/hr)
5. Haze, 23-km visibility
6. Very heavy rain (5 cm/hr)

A: The appropriate atmospheric windows are as follows:

| <u>Environmental Condition</u> | <u>Window</u> |
|--------------------------------|-------------------------|
| 1 | d |
| 2 | c |
| 3 | e (<i>marginally</i>) |
| 4 | g |
| 5 | b |
| 6 | h |

18. What is meant by Rayleigh scattering? Give an example.

A: Rayleigh scattering refers to the scattering of electromagnetic energy by particles which are small with respect to the wavelength of the scattered energy. The amount of scattering is proportional to the fourth power of the frequency. An example is the scattering of visible solar radiation by air molecules (leading to the reddish appearance of the sun at rising and setting, and to the blueness of the sky).

Chapter Five

19. This exercise refers to Figs. 3.5-1 and 3.5-3, and involves two objects:

- Hot metal housing surrounding a tank exhaust vent; temperature, 700 K; IR reflectivity, 0.4
- Black asphalt road; temperature, 270 K; IR reflectivity, 0.05

For each material, find the wavelength at which the peak of its thermal radiation curve occurs. Find the total radiated energy (watt/m³) at the wavelength of peak emission. Select a detector material that would be most suitable for sensing the radiation from each of the objects.

A: The results are summarized as follows:

| OBJECT | PEAK WAVELENGTH | PEAK ENERGY | DETECTOR MATERIAL |
|--------------|--------------------|---------------------------------------|-------------------|
| Hot metal | 4.3 μm | 2.6×10^9 watt/m ³ | Ge:Au or HgCdTe |
| Asphalt road | 11.1 μm | 3.5×10^7 watt/m ³ | HgCdTe |

Chapter Six

20. This exercise refers to a spaceborne millimeter-wave sensor (140 GHz) for passive detection of atmospheric vehicles from radiometric contrast temperature measurements.

- a. Assuming an orbital altitude of 800 km, compute the diameter of the antenna aperture required for resolution of 5-meter target objects.

- b. If the sensor were replaced by one operating at $10 \mu\text{m}$, what would be the required antenna aperture for resolution of a 5-meter object?

A: a. The angular resolution, in radians, is given by λ/D . At the stated frequency of 140 GHz, it would be necessary to have

$$D = 341 \text{ m}$$

to satisfy the stated resolution requirements. In effect, the result is that a millimeter-wave sensor at 140 GHz could not resolve objects as small as 5 m.

- b. If λ is changed to $10 \mu\text{m}$, then the required resolution is obtained for

$$D = 1.6 \text{ m}$$

a requirement that could be met in practice.

UNIT FOUR
REVIEW EXERCISES

Chapter Two

1. Explain what is meant by the term telemetry or telemetry data.

Answer: Telemetry refers to the transmission of instrument readings (measurements) of all kinds -- for example, temperature, pressure, or acceleration -- to an observer located at a remote site.

2. Assume that a spacecraft has the form of a circular cylinder 1 m in diameter and 100 m long. The spacecraft is aligned with the axis of the cylinder pointing toward the center of the earth, and is in a circular orbit at a height of 600 km. Compute the difference in gravitational attraction between the two ends of the cylinder.

Answer: Assuming an inverse square gravitational law, the difference in gravitational attraction is

$$\delta g = \frac{GM}{(R+h)^2} - \frac{GM}{(R+h+\ell)^2}$$

where

GM is the geocentric gravitational constant
(Table 1.3-2, Unit Two)

R is the radius of the earth

h is the satellite's height above the earth

ℓ is the length of the satellite

For simplicity, substitute

$$\rho = R+h$$

$$x = \frac{\ell}{\rho}$$

and use the binomial theorem to obtain a series expansion for δg in terms of the small quantity x . The result is

$$\delta g = \frac{2GMx}{\rho^2} + \text{terms of higher order}$$

Using the approximate numerical values

$$GM = 4.0 \times 10^{14} \text{ m}^3/\text{sec}^2$$

$$\rho = R+h = 7 \times 10^6 \text{ m}$$

$$x = \frac{\ell}{\rho} \approx 1.4 \times 10^{-5}$$

the answer is

$$\delta g = 2.3 \times 10^{-4} \text{ m/sec}^2 = 23 \text{ mgal}$$

3. If an earth satellite has no internal heat sources, will it eventually cool to a temperature of absolute zero?

Answer: No. It will reach an equilibrium temperature at which the solar energy absorbed by the spacecraft equals the infrared energy emitted by the satellite. The value of the equilibrium temperature depends on the reflective and absorptive qualities of the satellite's surface, but will probably be in the range of 200 to 250 K for a satellite near the earth.

Chapter Three

4. Assume a satellite in a retrograde, nearly polar, circular orbit with an inclination of 99 deg. The height of the satellite at its equatorial crossings is 890 km. Compute the daily change in Ω , the right ascension of the ascending node.

Answer: Using 6378160 m for the earth's equatorial radius, the satellite's semi-major axis is 1.1395. From Eq. (4.3-17) the node advances by 0.9864 deg/day.

5. For the satellite described in the preceding exercise, what is the annual nodal change? Why is such a satellite called sun-synchronous? Why are sun-synchronous satellites important for photographic and remote-sensing applications?

Answer: The annual change in Ω is very nearly 360 deg. The plane of the satellite's orbit rotates (with respect to a geocentric inertial coordinate system) at the same rate as the apparent motion of the sun -- hence the name sun-synchronous. In practice, this means that if the satellite crosses the equator on a northbound pass at local noon, it will make every northbound pass at local noon. (Southbound passes will be made, of course, at local midnight.) The significance for photographic and remote-sensing applications is that the satellite observes the earth, at every pass, under virtually identical conditions of solar illumination.

6. Could a sun-synchronous system be designed involving a satellite in a direct, rather than retrograde, orbit?

Answer: No. Examination of Eq. (4.3-17) shows that the required positive nodal rate can be obtained only if $\cos i$ is negative, which implies a retrograde orbit ($i > 90$ deg).

Chapter Four

7. What are some of the means of attitude determination used by satellites?

Answer: The student's answer should include:

- Sun sensors
- Horizon sensors
- Magnetometers (to detect spacecraft orientation relative to the earth's magnetic field)
- Star sensors
- Inertial reference systems.

8. Could a star tracker be used to track planets instead of stars? Which planets would be suitable? What would be the advantages and disadvantages of tracking a planet instead of a star?

Answer: Planets can be used, in principle. Most suitable planets are Jupiter, Saturn, and Venus (when its angular distance from the sun is great enough). The main advantage is that the planet is a very bright and easily identified target. Disadvantages include geometric limitations (the bright planets all lie very nearly in the same plane) and the computational burden associated with determining the planet's position as a function of time.

Chapter Five

9. Distinguish between active and passive systems for tracking satellites and determining their orbits.

Answer: The active systems transmit energy (radio, radar, laser) to the satellite, resulting in reflection or retransmission back to the ground. Passive systems do not transmit, but receive energy transmitted by the satellite or reflected from it (as in the case of sunlight). When the terms active and passive are applied to the satellite, rather than the tracking system, the distinction is between satellites that have onboard transmitters and those that do not.

10. Discuss the similarities and differences between laser and radar systems.

Answer: In principle, they are the same, differing only in the wavelength of the energy used. A laser is a radar operating in the visible (or infrared), rather than at so-called radar frequencies. There are practical distinctions in terms of power supply, instrumentation, and mounting requirements. Also, lasers are usually restricted to clear-weather operation.

11. Reference is made, in this and other chapters, to radar devices operating at C-band, L-band, S-band, etc. To what do these letters refer?

Answer: An old designation for radar frequencies, going back to World War Two security requirements, used code letters like C-band, L-band, etc. Although this system has been officially obsolete for many years, it is still used. An overview of the system:

| BAND | FREQUENCY LIMITS (GHz) | WAVELENGTH LIMITS (cm) |
|----------------|---------------------------|---------------------------|
| P | 0.3 to 1.0 | 30.0 to 100.0 |
| L | 1.0 to 2.0 | 15.0 to 30.0 |
| S | 2.0 to 4.0 | 7.5 to 15.0 |
| C | 4.0 to 8.0 | 3.75 to 7.5 |
| X | 8.0 to 12.5 | 2.4 to 3.75 |
| K ^u | 12.5 to 18.0 | 1.67 to 2.4 |
| K | 18.0 to 26.5 | 1.1 to 1.67 |
| K _a | 26.5 to 40.0 | 0.75 to 1.1 |

12. The orbital inclinations of the GEOS-3 and SEASAT-1 satellites are given in the text as 115 deg and 108 deg, respectively. What is the meaning of an inclination greater than 90 deg?

Answer: Inclinations greater than 90 deg indicate a retrograde orbit. This means that the motion of the satellite, as viewed by a hypothetical observer located above the north pole, is clockwise. If satellite motion is counterclockwise (in the same sense as the rotation of the earth) the orbit is called direct. (The term prograde is sometimes seen instead of direct, but this is an incorrect usage and should be avoided.)

13. Use Eq. (4.5-1) to estimate the velocity of an ocean current crossed at right angles by the subtrack of a satellite equipped with a radar altimeter, given that the altimeter detected a change in ocean height of 1 m over a 300 km current width. Assume that the crossing occurred at latitude 30 deg South.

Answer: Using these approximate values for the physical constants

$$g = 9.8 \text{ m/sec}^2$$

$$\omega = 7.3 \times 10^{-5} \text{ rad/sec}$$

and the given data

$$\Delta h = 1 \text{ m}$$

$$\phi = 30 \text{ deg}$$

$$\alpha = 90 \text{ deg}$$

$$\Delta L = 3 \times 10^5 \text{ m}$$

the use of Eq. (4.5-1) gives

$$V = 0.45 \text{ m/sec}$$

Velocities of ocean currents are often expressed in knots.
The conversion is

$$1 \text{ knot} = 0.5144 \text{ m/sec}$$

Thus, the computed velocity may also be given as

$$V = 0.87 \text{ knot}$$

14. Explain the meaning of the word ephemeris.

Answer: The original meaning of the word ephemeris (plural ephemerides) was much the same as diary or journal, referring to the recording of information on a daily basis. Astronomers used the word for tabulations, on a daily basis, of the positions of sun, moon, planets, and stars, as well as other time-varying astronomical data. Current usage no longer requires that the tabulation be daily, but applies to the representation of time-varying information in general, whether by listing or by formula. In the special sense of a satellite ephemeris, the meaning is a representation (by formula, tabulation, or otherwise) of the position (often, position and velocity) components as functions of time.

15. Using the data given in Table 4.5-1, calculate the orbital eccentricity for each of the three satellite types listed.

Answer: The given data are not sufficient for a precise answer, since the latitude of perigee and apogee is not given. As a result, the proper value of R , the earth's radius, in the formula for perigee and apogee altitudes:

$$h_a = a(1+e) - R$$

$$h_p = a(1-e) - R$$

is not known. In these formulas, a is the semi-major axis and e is the eccentricity. Using an average value for R ,

$$R = 6.37 \times 10^6 \text{ m}$$

(technically, this is the radius of a sphere having the same volume as the earth), and the data from Table 4.5-1, the results are:

| SATELLITE TYPE | ECCENTRICITY |
|-------------------|--------------|
| Navy Navigation | 0.00 |
| High Eccentricity | 0.75 |
| Low Perigee | 0.37 |

The computation is done most easily by using the formulas in the form:

$$a = R + \frac{h_a + h_p}{2}$$

$$e = \frac{h_a - h_p}{2a}$$

Chapter Six

16. If a satellite is transmitting at a frequency held precisely at 150 MHz, and is approaching an observer at 100 m/sec, find the frequency of the signal received by the observer.

Answer: The Doppler shift increases the received frequency by 50 Hz (Eq. 4.6-2); the signal as received by the observer has a frequency of 150,000,050 Hz.

17. How accurately must a ground observer measure the frequency of the signal received from a satellite in order to determine the radial velocity to within one mm/sec? Assume that the satellite transmitter frequency is 400 MHz.

Answer: From Eq. (4.6-2) it is seen that the relation between a small error in the received frequency, δf_r , and a small error in the radial velocity, δs , is given by

$$\delta s = \frac{c}{f_t} \delta f_r$$

With the given values

$$f_t = 4 \times 10^8 \text{ Hz}$$

$$\delta s = 10^{-3} \text{ m/sec}$$

and using

$$c = 3 \times 10^8 \text{ m/sec}$$

for the velocity of light, the result is

$$\delta f_r = 0.0013 \text{ Hz}$$

This level of accuracy would be very difficult to achieve in practice.

18. During a tracking interval of 10 sec, the frequency of the signal received from a satellite increases linearly from

$$f_r(0) = f_{ref} = 150 \text{ MHz}$$

to

$$f_r(10) = f_{ref} + 50 \text{ Hz}$$

Find the total Doppler count over this interval.

Answer: Using Eq. (4.6-4), the integrand, $f_r - f_{ref}$, can be expressed as

$$f_r - f_{ref} = 5t$$

where t , the time measured from the beginning of the tracking period, varies from 0 to 10 sec. Thus, Eq. (4.6-4) becomes

$$N_{12} = \int_0^{10} (5t) dt$$

and N_{12} , the Doppler count, is 250 cycles.

19. Assume that a ground receiver measures a Doppler count of 250 cycles during a tracking interval of 10 sec. The receiver's stabilized reference frequency is exactly 150 MHz, while the satellite's transmission frequency is known to be 149.998 MHz. Compute the change in range (from satellite to receiver) during the tracking interval.

Answer: From Eq. (4.6-9), the change in range is computed to be

$$\delta s = -0.1 \text{ mm}$$

For all practical purposes, the change in range is zero. This result illustrates the fact that the measured Doppler count has two sources:

- The Doppler shift caused by relative motion
- The difference between transmitter and reference frequencies ($\delta f = f_t - f_{ref}$).

In this example, the observed Doppler count arises almost entirely from the difference in frequencies, δf .

20. Discuss the accuracy requirements for measured Doppler count in terms of desired accuracy in the determination of change in range.

Answer: Equation (4.6-9) shows that small errors in the change in range, δs , are related to errors in the Doppler count, N_{12} , by the factor c/f_{ref} . For the NNSS frequencies, this factor is roughly unity; thus, errors in N_{12} (measured in cycles) translate directly into errors in δs (measured in meters). For example, an accuracy level of 0.01 cycle corresponds to 1 cm of range difference.

21. Why does the NNSS use the dual-frequency (150 and 400 MHz) mode of operation?

Answer: The use of two frequencies permits compensation for shifts in frequency caused by ionospheric effects. Without this compensation, the accuracy of the system would be degraded to an unacceptable level.

Chapter Seven

22. Why is GPS sometimes referred to as a four-dimensional positioning system?

Answer: The terminology reflects the fact that GPS provides the user with three-dimensional position information, as well as precise time.

23. Explain the meaning of the term pseudo-range, as used in the context of GPS.

Answer: GPS users derive range information from measured time-of-arrival (TOA) information. Because of uncertainty in the user's knowledge of GPS system time, there is a bias error in the derived range. The range information which contains this bias error is called pseudo-range.

24. Would the predicted accuracy of the fully operational GPS be adequate for use by military aircraft as a precision landing system under conditions of zero ceiling and visibility?

Answer: No. This is not a projected application of GPS.

Chapter Eight

25. Why are remote-sensing satellites like LANDSAT placed in nearly polar orbits?

Answer: To permit nearly global geographic coverage. For another important reason, review Exercises 4, 5, and 6.

26. Which agencies of the United States government are primarily involved in the processing and dissemination of LANDSAT data?

Answer: The U.S. Geological Survey (under the Department of the Interior) and NASA -- primarily the Goddard Space Flight Center -- share responsibility for the processing and dissemination of LANDSAT data. Design, launch, and control are a NASA responsibility.

27. Identify the three map projections used for the presentation of LANDSAT data in map-compatible form.

Answer: The polar stereographic projection is preferred for latitudes higher than 65 deg. The projections used at lower latitudes are the Universal Transverse Mercator (UTM) and the Space Oblique Mercator (SOM). The first two of these are discussed in Section 1.2.6 of Unit One. A detailed discussion of the SOM is beyond the scope and level of these Lecture Notes; interested students may be referred to the original sources:

- "Space Oblique Mercator" by A.P. Colvocoresses, published in Photogrammetric Engineering, Volume 40 (1974), pages 921 to 926.
- "The Space Oblique Mercator Projection" by J.P. Snyder, published in Photogrammetric Engineering and Remote Sensing, Volume 44 (1978), pages 585-596.

The following simplified explanation may be helpful. The ordinary Mercator Projection involves a projection from the globe onto a cylinder whose axis coincides with the earth's spin axis. Globe and cylinder are tangent along the equator. In a Transverse Mercator Projection, the cylinder is tilted 90 deg, so that its axis is perpendicular to the earth's spin axis. Globe and cylinder are tangent along a meridian. For the Oblique Mercator Projection, the cylinder is tilted at an intermediate angle and is tangent along some great circle. If this great circle is aligned with the subtrack of a satellite, the resulting projection is a first approximation to the Space Oblique Mercator. Further stages of mathematical refinement are required because the actual satellite subtrack is not exactly a great circle. The deviations arise from

- Rotation of the earth
- Precession of the satellite's orbital plane
- Other perturbations in the motion of the satellite.